An Adaptive Path Planning Method Based on the Frenét Frame*

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***Abstract*—In the realm of mobile robot path planning, this study integrates local path planning with the *Frenét Frame*, presenting an adaptive path planning approach. Firstly, the A* algorithm is employed to extract the shortest path from the map. Subsequently, cubic spline interpolation is applied to the path to generate a smooth global reference trajectory. Next, a cost function based on lateral offset distance is introduced to enhance the proximity of the trajectory to the reference trajectory, thereby improving efficiency. Lastly, an adaptive sampling strategy is incorporated to enhance the robot's dynamic obstacle avoidance performance. Experimental results validate the efficacy of the proposed path-planning approach.**

***Keywords*—path planning, the Frenét Frame, adaptive strategy**

I. Introduction

A mobile robot [1] is an autonomous machine capable of navigating complex environments, encompassing functions including perception, decision-making, control, and execution [2]. Path planning [3] for mobile robots is a prominent research subject extensively investigated within the realm of robotics. It holds pivotal significance in augmenting the intelligence and efficiency of mobile robots [4]. The prevailing categorization involves two fundamental tiers: global path planning and local path planning [5]. Global path planning pertains to the identification of an optimal or near-optimal trajectory between an initial point and a target destination on a map. Prominent algorithms for global path planning encompass the A* algorithm [6], Dijkstra's algorithm [7], the D* algorithm [8], and the Rapidly-Exploring Random Tree (RRT) [9] algorithm. Aiming at the low search efficiency of the A* algorithm, a bidirectional adaptive A* algorithm is proposed in [10], which adopts a directional search strategy to improve the expansion efficiency, and uses adaptive step size and adaptive weight strategy to improve the search speed of the algorithm. In [11], given the shortcomings of the child node search strategy of the A* algorithm, the jump point search algorithm (JPS) was proposed, which effectively improved the path search efficiency by filtering jump points. Nevertheless, it's worth noting that global path planning approaches often center their attention solely on the overall path connecting the origin and destination points, potentially neglecting local contextual shifts experienced by the robot along the course [12]. Consequently, during the execution of a globally devised trajectory, the robot may encounter challenges in adapting to local impediments, topographical variations, and dynamic alterations. This underscores the need for local path planning to enable real-time maneuvering under such circumstances. Within the domain of local path planning, the principal objective involves the identification of a secure and efficient path within the immediate local environment to accommodate robots. This strategy empowers them to fulfill the overarching goals delineated by the comprehensive path-planning endeavor. Widely employed techniques for local path planning encompass the Dynamic Window Approach (DWA) [13], Timed Elastic Band (TEB) [14], and Model Predictive Control (MPC) [15], among others. These methods also suffer from issues such as high computational complexity, limited foresight, subpar dynamic obstacle avoidance effectiveness, and non-global optimality.

This paper amalgamates robotic path planning with the Frenét Frame, presenting an adaptable path planning technique grounded in the Frenét Frame. The primary contributions of this study are outlined as follows: 1) Utilization of the partitioned optimal path intervals, as obtained through the A* algorithm, as the reference line within the Frenét Frame after cubic spline interpolation. This enhancement fosters road smoothness while preserving asymptotic path optimality. 2) Proposal of a cost function contingent on horizontal planning distance for path planning within the Frenét Frame, aimed at maintaining path proximity to the reference line and elevating travel efficiency. 3) An adaptive sampling approach is introduced, enabling real-time adjustments to sampling distance and intervals when obstacles manifest along the reference line.

This work was supported in part by the National Key Research and Development Program of China under Grant 2021ZD0114503, the Major Research plan of the National Natural Science Foundation of China under Grant 92148204, the National Natural Science Foundation of China under Grants 62027810 and 61971071, Hunan Leading Talent of Technological Innovation under Grant 2022RC3063, Hunan Science Fund for Distinguished Young Scholars under Grant 2021JJ10025, Hunan Key Research and Development Program under Grants 2021GK4011, 2022GK2011, Changsha Science and Technology Major Project under Grant KH2003026, Joint Open Foundation of State Key Laboratory of Robotics under Grant 2021-KF-22-17, China University Industry-University-research Innovation Fund under Grant 2020HYA06006.

This innovation contributes to the advancement of dynamic obstacle evasion efficacy.

II. Global Path Planning

The A* algorithm is presently considered the most efficient direct search method for solving optimal paths in static maps. It integrates elements of both breadth-first and depth-first approaches, constituting a heuristic search algorithm. In the context of this research, the A* algorithm is also employed for generating global paths. The A* algorithm assesses the merits of nodes using a cost function, wherein the cost function is expressed as:

$$f(n) = g(n) + h(n) \tag{1}$$

Where $f(n)$ signifies the estimated cost from the starting point to the endpoint traversing node n. $g(n)$ represents the actual cost, i.e., the factual distance covered from the starting point to node n. Each search iteration is consistently updated to a shorter distance, ensuring optimality from the origin to node n. $h(n)$ denotes the projected cost, characterizing the anticipated direct distance between node n and the endpoint. After deriving an optimal path from the initial to the terminal point via the A* algorithm, a cubic spline interpolation is applied to smooth the optimal path, thereby obtaining the reference trajectory for subsequent local path planning. The reference trajectory is partitioned into multiple small intervals, with each interval being subjected to cubic spline interpolation. The functional representation of cubic spline interpolation is given by:

$$S_i(x) = a_i + b_i x + c_i x^2 + d_i x^3 \tag{2}$$

Where a_i, b_i, c_i, d_i are the variables to be determined. The resulting approximated optimal path after interpolation becomes the reference trajectory for local path planning.

III. Adaptive Path Planning Based On The Frenét Frame

A. Algorithm Principle

The Frenét Frame [16] is a frequently employed coordinate system for delineating curved motion. It delineates positional deviation from a reference line through longitudinal distance s and lateral distance d. The utilization of the Frenét Frame in path planning facilitates the division of the two-dimensional motion quandary into two distinct one-dimensional optimization predicaments, thus simplifying the complexities tied to path planning and control. Path planning method based on the Frenét Frame, as proposed by Werling [17]. The high-dimensional optimization challenge of motion planning is dissected into two autonomous optimization predicaments along the horizontal and vertical axes using the Frenét Frame. This approach accomplishes the separation of the motion planning predicament and thereby mitigates the intricacy associated with planning.

$$L(s(t), d(t)) = \vec{t}_r(s(t)) + \vec{n}_r(d(t)) \tag{3}$$

Where $L(s(t), d(t))$ is the original high-dimensional optimization problem, and $\vec{t}_r, \vec{n}_r$ represent the tangent vector and normal vector, respectively, corresponding to a specific point on the reference line during planning. The horizontal and vertical independent optimization problems are derived utilizing these two vectors. In the realm of autonomous driving, given the relative absence of pronounced fluctuations in vehicle lateral and longitudinal speed and acceleration as perceived by humans, the concept of jerk(J) is introduced to gauge the quality of the planned path. Given that the path planning is partitioned into horizontal and vertical dimensions, the minimization of the loss function can also be expanded separately within these dimensions. In other words, given the time interval $T := t_1 - t_0$, the initial state $P_0 = [p_0, \dot{p_0}, \ddot{p_0}]$, and the target state $P_1 = [p_1, \dot{p_1}, \ddot{p_1}]$, find the path $p(\tau)$ corresponding to the minimum cost $J_t(p(t))$:

$$J_t(p(t)) := \int_{t_0}^{t_1} \dddot{p}^2(\tau) d\tau \tag{4}$$

In [18], it has been demonstrated that solutions to any jerk optimization problem can be accurately represented using a quintic polynomial. Thus, for lateral motion, a quintic polynomial interpolation method is employed. By specifying lateral sampling range $[d_{\min}, d_{\max}]$, prediction time $[T_{\min}, T_{\max}]$, and sampling interval $[\triangle d, \triangle t]$, a set of planning trajectory curves can be determined for lateral motion. The lateral cost associated with each alternative curve is calculated by designing a loss function tailored to lateral motion. Similarly, for longitudinal motion, a quartic polynomial interpolation method is utilized. By providing the *target speed* for longitudinal sampling, prediction time $[T_{\min}, T_{\max}]$, and sampling interval $[\triangle v, \triangle t]$, multiple planning trajectory curves for longitudinal motion are derived. The longitudinal cost of each candidate curve is computed using a loss function specifically designed for longitudinal motion. Finally, by comprehensively considering both lateral and longitudinal motion costs, an integrated approach yields a locally optimal trajectory.

B. Cost Function Based on Lateral Planning Distance

To ensure that the robot can traverse along the optimal path obtained from the A* algorithm, this study presents a cost function $Cost(J_d)$ based on the lateral displacement d within the context of the Frenét Frame. $Cost(J_d)$ considers the lateral offset between the robot's present position and the reference trajectory as a relatively significant cost factor. Its purpose is to confine the robot's motion to remain close to the reference trajectory during the process of local path planning. This constraint aims to enhance the efficiency and precision of the overall motion planning. The cost function for lateral distance is given as follows:

$$\begin{aligned} Cost(J_d) = \alpha \cdot \sum_{n=0}^{T_i} d_i^2 + \beta \cdot \sum_{n=0}^{T_i} (path\_curvature_i)^2 \\ + \gamma \cdot T_i \end{aligned} \tag{5}$$

Where d is the lateral displacement of each point along the path concerning the reference trajectory, $path\_curvature_i$ represents the curvature value corresponding to each point on the path, and T_i stands for the current path's prediction time. The minimum prediction time is 1.0 seconds, and the maximum prediction time is 2 seconds. The coefficients α, β, γ are utilized as weighting factors to balance the impact of lateral displacement, path curvature, and the expected planning time on the path planning quality. Upon completing a round of path planning, the selection of an optimal path for advancement and subsequent rounds of path planning is necessary until reaching the destination point or encountering an impasse. The filtering criteria for the optimal path, denoted as $best\_path$, in each round are defined as follows:

$$\begin{aligned} P_{\underset{\min}{cost}} = \min(k_d \cdot Cost(J_d) + k_v \cdot ((target\_speed \\ - v_s)^2 + T_i)) \end{aligned} \tag{6}$$

Where P_{cost} represents the path cost, and *target_speed*, v_s are the desired speed and planned speed respectively. For each path that meets the criteria, the cost is calculated, and the path with the lowest cost is chosen as the optimal path for the current round of path planning.

C. Adaptive Sampling Strategy

In traditional local path planning based on the Frenét Frame, both lateral and longitudinal sampling distances are often fixed. However, in the presence of dynamic obstacles, fixed sampling distances might lead to the robot's inability to maneuver promptly, resulting in unexpected situations. To address this issue, this paper proposes an adaptive planning distance adjustment strategy, building upon the previously defined cost function for lateral planning constraints.

$$d = \begin{cases} \frac{4D}{1+e^{0.5(s-3D)}}, & \text{if } D \le s < 5D \\ \frac{4D}{1+e^{0.5(s-7.5D)}}, & \text{if } 5D \le s < 10D \\ D, & \text{otherwise} \end{cases} \tag{7}$$

Where D represents the collision detection radius of the robot, s denotes the distance between the robot and the obstacle along the current path, and d signifies the planning radius obtained from the adaptive strategy. When there are no obstacles on the reference trajectory, the local path planning employs a smaller lateral planning distance to suppress excessive vehicle deviation from the reference trajectory. To enhance path planning robustness and foresight, a longer longitudinal planning distance is used. By increasing the longitudinal planning distance, the robot can anticipate future road condition changes over a greater distance, facilitating faster obstacle detection and ensuring the robot's movement remains as close to the reference trajectory as possible, thereby enhancing efficiency.

When the local path planning system anticipates the presence of dynamic obstacles on the reference trajectory, it triggers the corresponding adaptive planning strategy. In such scenarios, the algorithm progressively increases the lateral planning distance while reducing the longitudinal planning distance. This adjustment aims to find a suitable path that can navigate around the obstacles. Once such a path is found, the algorithm swiftly readjusts its strategy, employing a smaller lateral planning distance to ensure the robot promptly returns to the vicinity of the reference trajectory, maintaining overall movement efficiency. Simultaneously, it moderately increases the longitudinal planning distance to ensure the robot smoothly returns to the reference trajectory after bypassing obstacles, thus enhancing overall path planning smoothness and robustness. The entire path-planning process of the adaptive strategy is illustrated in Fig.1.

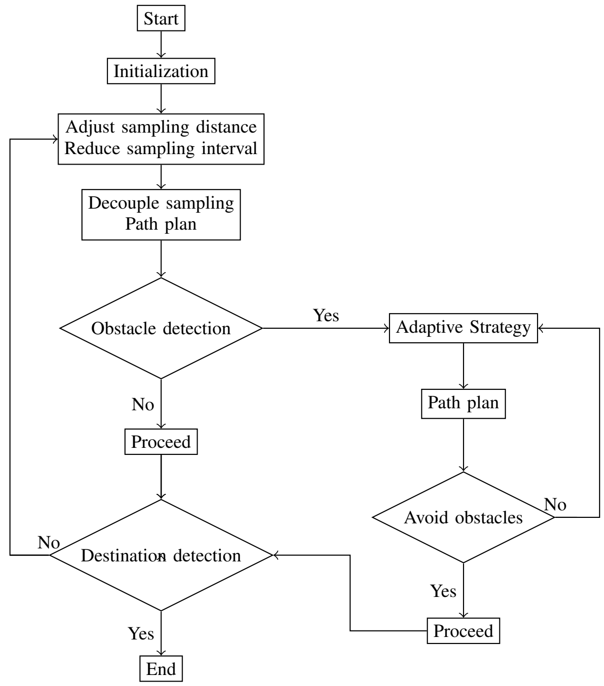


Fig. 1. Flowchart depicting path planning integrating adaptive strategy and cost function

IV. EXPERIMENT RESULTS

To validate the effectiveness of the proposed algorithm in this study, comparative experiments were conducted between the Werling method and the method presented in this paper. Both methods utilized the smoothed approximate optimal path obtained using the A* algorithm as the reference line.

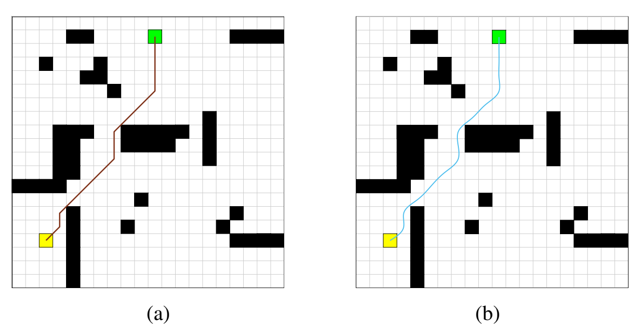


Fig. 2. (a) represents the optimal path obtained through the A* algorithm, while (b) depicts the asymptotically optimal path after undergoing smoothing procedures.

The optimal path was extracted from the map using the A* algorithm. Subsequently, a three-order spline interpolation was applied to the path followed by smoothing, resulting in an asymptotically optimal trajectory. This trajectory serves as the reference guide for subsequent local path planning, as illustrated in Fig.2. The starting point is denoted by a yellow square at coordinates (3, 4), while the endpoint is represented by a green square at coordinates (11, 19).

A. The reference trajectory is obstacle-free

In this context, a comprehensive comparative analysis was conducted on planning time, planning frequency, and the quality of the resultant path. The starting point of local planning was established at a position in the Frenét Frame with a lateral deviation of 0.5 and a longitudinal deviation of 0, denoted as (0.5, 0). The initial velocity was set to 12 km/h, with both lateral acceleration and jerk set to 0.

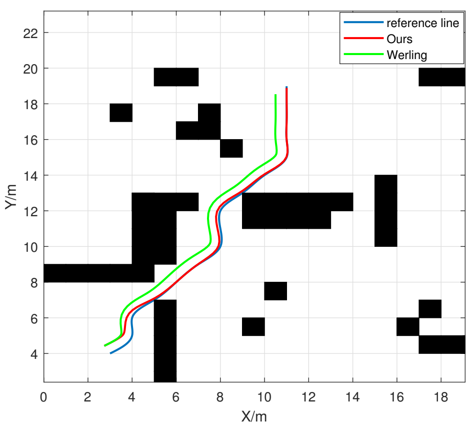


Fig. 3. Depict the final paths obtained after the completion of path planning, with the reference trajectory shown in blue, the proposed method in red, and the Werling method in green.

TABLE I
ALGORITHM COMPARISON

Algorithm name	Running times	Planning Frequency	RMSE
Werling	11.547	88	0.50079
Ours	**5.814**	**74**	**0.19846**

In the case where the starting point is (0.5, 0), the result of the Werling method is a path that is approximately parallel to the reference line but does not closely follow it, in Fig.3. This is mainly because the Werling method does not consider the lateral deviation in the Frenét Frame as one of the criteria for path planning. In contrast, in our approach, after incorporating a cost function related to the lateral deviation, the path can closely approach the reference line with enhanced speed. Moreover, favorable outcomes are achieved in terms of path deviation, execution time, and the number of planning iterations, in Table.I.

B. The reference trajectory is obstructed by obstacles

This experiment introduces obstacles along the reference line to simulate situations where obstacles may obstruct the path during traversal in the same map scenario. Figure 4 illustrates the experimental setup with an obstacle placed at (10, 14) on the reference line.

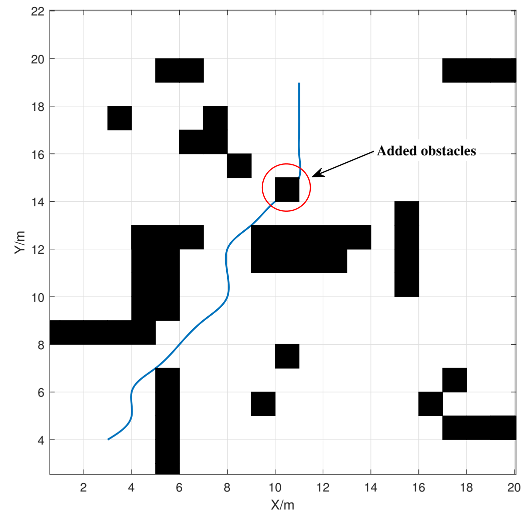


Fig. 4. Map after adding obstacles

From Fig. 5(a), it is evident that the Werling method encounters path planning failure when faced with obstacles on the reference line due to insufficient lateral planning distance. As a consequence, all planned paths collide with the obstacles. In contrast, the approach proposed in this paper capitalizes on the benefits of an adaptive planning strategy. This strategy expands the sampling range upon obstacle detection, resulting in the successful design of a safe path that circumvents obstacles, as shown in Fig.5(b).

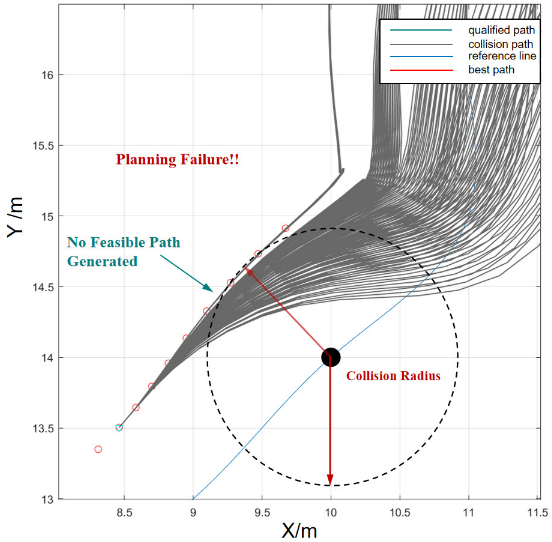
TABLE II
ALGORITHM COMPARISON

Algorithm name	Running times	Planning Frequency	RMSE
Werling	failed	failed	failed
Ours	**6.092**	**76**	**0.46578**

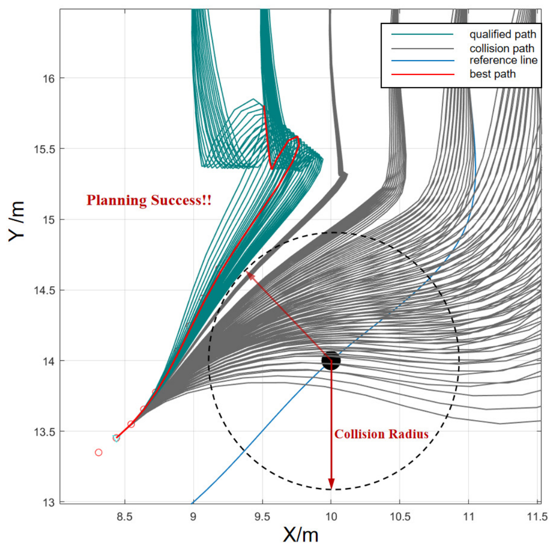
Subsequently, empowered by the synergistic effects of the adaptive sampling strategy and the cost function predicated on lateral deviation distance, the robot is endowed with the capability to autonomously strategize an optimal trajectory, seamlessly realigning with the reference line and culminating in a successful termination at the destination point. The consequential outcomes of the path planning endeavor are meticulously tabulated and delineated in TableII.

V. CONCLUSION

This paper presents an adaptive planning algorithm based on the Frenét Frame, suitable for the domain of mobile robotics. In terms of reference line selection, the optimal path obtained from the A* algorithm is subjected to a smoothing process to derive an approximate optimal reference line. Additionally, a cost function dependent on lateral deviation distance is introduced to govern the robot's trajectory. Addressing potential



(a)



(b)

Fig. 5. (a) and (b) represent the scenarios when obstacles are present on the reference line, comparing the Werling method and the method proposed in this paper, respectively. Solid black dots represent obstacles, and black dashed circles at a fixed distance from the obstacles depict the collision zones, with a region radius denoted as the Collision Radius. The red curves in the figures signify the optimal paths, the gray curves represent collision-encountering infeasible paths, and the blue curves represent qualified paths.

dynamic obstacles on the reference line, an adaptive sampling strategy is proposed, facilitating obstacle avoidance through the adjustment of sampling distances under varying circumstances. Through empirical validation, the effectiveness of the proposed approach is evidenced in terms of planning time, number of planning iterations, and final trajectory deviation.

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